

Citation for published version:

Carley, M 2010, 'The near-field of spinning sources: Why source identification is hard', Paper presented at 159th Meeting of the Acoustical Society of America/NOISE-CON 2010, Baltimore, USA United States, 19/04/10 - 23/04/10 pp. 040004-040004-11. <https://doi.org/10.1121/1.3431358>

DOI:

[10.1121/1.3431358](https://doi.org/10.1121/1.3431358)

Publication date:

2010

[Link to publication](#)

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An asymptotic analysis is presented for the near field of spinning sources, based on a transformation into cylindrical coordinates centred on a line at a fixed radius from the source axis. This transforms the circular source into an equivalent finite length line source with a source distribution made up of ‘modes’ given as Chebyshev polynomials of the second kind $U_n(s) = \sin[(n+1) \cos^{-1} s] / \sin \cos^{-1} s$. These ‘modes’ play a role like that of modes in ducts and the analysis shows that their acoustic field propagates or decays depending on whether $n+1 < k$ or $n+1 \geq k$, respectively, with k wavenumber, similar to the cut-on/cut-off behavior of duct modes.

The analysis is used to examine the problem of identifying a source from field measurements. This has a wide range of applications and is recognized to be (very) ill-conditioned. Using the information supplied by the analysis of the source near field, the reasons for this ill-conditioning are explained.

1 Introduction

A widely-studied, hard problem is that of identifying the source distribution responsible for a spinning sound field. This covers noise generated by rotating systems such as cooling fans [1, 2], helicopter rotors [3, 4] ducted systems [5–10] and by jets [11] if we consider a jet to be a distribution of disk-shaped sources. In each of these applications, the aim is to use measurements of the acoustic field to infer, in some sense, the source distribution responsible for the noise.

There are a number of reasons why an estimate of the source might be required. The first case is where the source itself is of interest. This might be where source terms correspond to some physical feature of the system being studied and the real interest is in these physical features, or, it might be where the acoustic source itself is required as a preliminary to applying noise reduction measures. Examples of these two cases include the determination of the pressure distribution on a wing from the radiated noise [12], and the determination of the acoustic sources to be used as input to a noise control system for cooling fans [1, 2]. This has been recognized as a very ill-conditioned problem, with a number of solution techniques proposed to circumvent the difficulties. The issue is that the acoustic field around subsonic spinning sources decays exponentially with distance from the source [13–16], meaning that projecting the field backwards onto the source involves errors which increase exponentially.

The second case is where an estimated source will be used in predicting the field at points other than the original measurement positions. One example is the extraction of source parameters from the near field of a propeller in a wind tunnel, with the parameters being used to predict the far field [17]. This is a rather different problem to the first, because in projecting the near field into the far field, errors decay exponentially for a spinning field so that the accuracy required of the estimated source is not as high as in the previous case.

The problem considered in this paper is that of the field radiated by a distribution of monopoles over a disk. This corresponds to the case of ‘thickness’ noise from an open rotor or sound generated by a baffled piston. It is also a good approximation to the sound radiated from a duct termination [18] over much of the forward arc, making it useful in examining

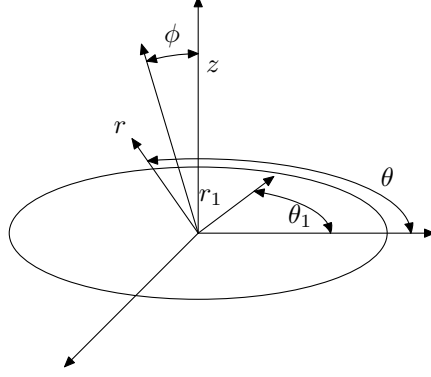


Figure 1: Coordinate system for radiation prediction

the problem of duct mode identification in aircraft engines. We further note that the dipole and quadrupole fields are found by differentiating the same basic integral [19, 20], so that conclusions drawn from analysis of the monopole problem will be applicable to the case of the higher order sources of importance in rotor acoustics and jet noise respectively.

2 Analysis

The problem considered is that of the field generated by an azimuthally varying distribution of monopoles with strength $s(r_1, \theta_1)$ given by the Rayleigh integral [13, 20]

$$p(r, \theta, z, \omega) = \int_0^1 \int_0^{2\pi} s(r_1, \theta_1) \frac{e^{jkR}}{4\pi R} d\theta_1 r_1 dr_1, \quad (1)$$

$$R^2 = r^2 + r_1^2 - 2rr_1 \cos(\theta - \theta_1) + z^2,$$

where the source is distributed over the unit disk in the plane $z = 0$, variables of integration have subscript 1 and the coordinate system is shown in Figure 1. The wavenumber $k = \omega/c$, and c is the speed of sound.

Taking one azimuthal mode of the source distribution, $s(r_1, \theta_1) = s_n(r_1) \exp jn\theta_1$, the radiated field for one mode can be written $p = p_n \exp jn\theta$:

$$p_n(r, z) = \int_0^1 \int_0^{2\pi} s_n(r_1) \frac{e^{j(kR - n\theta_1)}}{4\pi R} d\theta_1 r_1 dr_1, \quad (2)$$

$$R^2 = r^2 + r_1^2 - 2rr_1 \cos \theta_1 + z^2.$$

The integral of Equation 2 has been extensively studied due to its relevance to rotor acoustics and, under suitable conditions, as a good approximation to radiation from ducts. Many problems in source identification [1, 2, 5–10] can be viewed as attempts to recover the source term $s_n(r_1)$ from measurements of p_n .

In the remainder of the paper, we derive some basic properties of this integral which will show the fundamental limits placed on source identification methods by the sparse

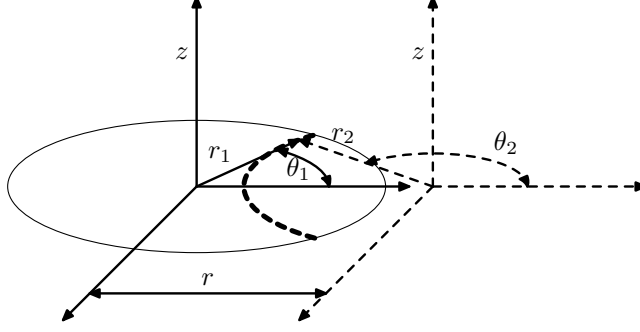


Figure 2: Coordinates for transformation to equivalent line source

information about the source which is radiated into the acoustic field, including the near field.

2.1 Equivalent line source

The first step in the analysis is to transform the disk source into a line source which generates (exactly) the same acoustic field. This is a transformation which has been used, in the axisymmetric case, in studies of transient radiation from pistons [21, 22], and with azimuthal variation in studies of rotor acoustics [13–16]. The first stage is to switch from the source-centred cylindrical coordinates (r, θ, z) of Figure 1 to the observer-centred coordinates (r_2, θ_2, z) of Figure 2. Under this transformation, Equation 2 becomes, for $r > 1$:

$$p_n(k, r, z) = \int_{r-1}^{r+1} \frac{e^{jkR}}{R} K(r, r_2) r_2 dr_2, \quad (3)$$

$$R = (r_2^2 + z^2)^{1/2},$$

$$K(r, r_2) = \frac{1}{4\pi} \int_{\theta_2^{(0)}}^{2\pi - \theta_2^{(0)}} e^{-jn\theta_1} s_n(r_1) d\theta_2, \quad (4)$$

where the source function $K(r, r_2)$ depends on r , the observer lateral separation, but is independent of z , the axial displacement. The coordinate systems are related by:

$$r_1^2 = r^2 + r_2^2 + 2rr_2 \cos \theta_2, \quad (5a)$$

$$\theta_1 = \tan^{-1} \frac{r_2 \sin \theta_2}{r + r_2 \cos \theta_2}. \quad (5b)$$

and the limits of integration in Equation 4 are given by setting $r_1 = 1$:

$$\theta_2^{(0)} = \cos^{-1} \frac{1 - r^2 - r_2^2}{2rr_2}. \quad (6)$$

The function $K(r, r_2)$ has square-root behaviour at its end-points, $r_2 = r \pm 1$, so that it can be expanded:

$$K(r, r_2) = \sum_{m=0}^{\infty} u_m(r) U_m(s) (1 - s^2)^{1/2} \quad (7)$$

with $s = r_2 - r$ and $U_n(s)$ the Tchebysheff polynomial of the second kind.

Inserting this expansion into Equation 3

$$p_n(k, r, z) = \sum_{m=0}^{\infty} u_m I_m(k, r, z), \quad (8)$$

$$I_m(k, r, z) = \int_{-1}^1 \frac{e^{jkR}}{R} U_m(s) (r + s) (1 - s^2)^{1/2} ds, \quad (9)$$

$$= \int_0^\pi \frac{e^{jkR}}{R} (r + \cos \beta) \sin(m+1)\beta \sin \beta d\beta, \quad (10)$$

$$R^2 = (r + \cos \beta)^2 + z^2,$$

with the transformation $s = \cos \beta$ and use of the definition of the Tchebysheff polynomial [23, 8.940.2], $U_m(s) = \sin[(m+1)\beta] / \sin \beta$.

3 Radiated field

The analysis of the previous section gives us a model of a spinning acoustic field expressed in terms of an exactly equivalent line source composed of a superposition of modes given as Tchebysheff polynomials. In this section, we use this model to draw basic conclusions about the acoustic information which is available for source identification.

3.1 Cut-off modes

The first conclusion we can draw from the integral expression for I_m is that modes with $m > k$ decay exponentially and can be considered ‘cut-off’. For $z = 0$, $I_m(k, r, 0)$ can be evaluated exactly using standard relations for Bessel functions [23, 8.411.1, 8.471.1] and the definition of the Tchebysheff polynomial above:

$$I_m(k, r, 0) = j^m \pi (m+1) \frac{J_{m+1}(k)}{k} e^{jkr}. \quad (11)$$

For $m+1$ large and $k < m+1$, the Bessel function $J_{m+1}(k)$ decays exponentially, i.e. the higher modes are ‘cut off’. Since $|I_m(r, z)|$ has its maximum in the plane $z = 0$, we can further conclude that modes with $m > k$ are cut off everywhere and cannot be detected in the field. This is an exact result which places a first limit on the information radiated.

3.2 Asymptotic analysis

A second limit on the information available in the acoustic field can be found by asymptotic analysis of I_m which can be rewritten:

$$I_m = (Q_{m+2}(k, r, z) + Q_{-m-2}(k, r, z) - Q_m(k, r, z) - Q_{-m}(k, r, z))/4,$$

with:

$$\begin{aligned} Q_m(k, r, z) &= \int_0^\pi e^{jk\psi(\beta)} \frac{r + \cos\beta}{R} d\beta, \\ \psi(\beta) &= R + \gamma\beta, \\ \gamma &= m/k. \end{aligned} \tag{12}$$

The integral Q_m is in a suitable form for stationary phase analysis [24], which depends on finding the stationary points of ψ where $d\psi/d\beta = 0$ with $0 \leq \beta \leq \pi$. Upon differentiation and rearrangement, the condition $d\psi/d\beta = 0$ takes the form of a quartic equation:

$$(\alpha^2 - C^2)(r + C)^2 - \gamma^2 z^2 = 0, \tag{13}$$

where $C = \cos\beta$ and $\alpha = (1 - \gamma^2)^{1/2}$. To lie in the domain of integration, the stationary phase points must be real with $|C| < 1$. This leads to the requirement that $0 < \gamma < 1$ and $|z| < z_c$, a ‘cut-off’ value beyond which the phase function ψ has no valid stationary points. The two values of C , denoted C_+ and C_- , at the limits of $|z|$ are:

$$C_\pm(z) = \begin{cases} \pm\alpha, & z = 0, \\ -r/4 + (r^2 + 8\alpha^2)^{1/2}/4, & z = \pm z_c. \end{cases} \tag{14}$$

Denoting $C_c = C_+(z_c) = C_-(z_c)$, the cut-off value of z is:

$$\begin{aligned} z_c &= (\alpha^2 - C_c^2)^{1/2}(r + C_c)/\gamma, \\ &\rightarrow \alpha r/\gamma, \quad \alpha/r \rightarrow 0. \end{aligned} \tag{15}$$

Written in spherical coordinates, the asymptotic cut-off lines $z_c = \alpha r/\gamma$ are rays with polar angle $\phi = \sin^{-1}\gamma$. For completeness, we note that for $\gamma = 0$, there is no cutoff and the line source mode radiates into the whole field with amplitude proportional to $k^{-1/2}$.

Using the previous results, the asymptotic behaviour of the basic integral is given by:

$$\begin{aligned} Q_m &\sim j^{3/2} \frac{e^{jk\psi_+}}{(kR_+)^{1/2}} \left[\frac{2\pi}{C_+(r + 2C_+) - \alpha^2} \right]^{1/2} (r + C_+) \\ &\quad + j^{1/2} \frac{e^{jk\psi_-}}{(kR_-)^{1/2}} \left[\frac{2\pi}{C_-(r + 2C_-) - \alpha^2} \right]^{1/2} (r + C_-), \quad k \rightarrow \infty, \\ R_\pm^2 &= (r + C_\pm)^2 + z^2, \quad \psi_\pm = R_\pm + \gamma \cos^{-1} C_\pm, \end{aligned} \tag{16}$$

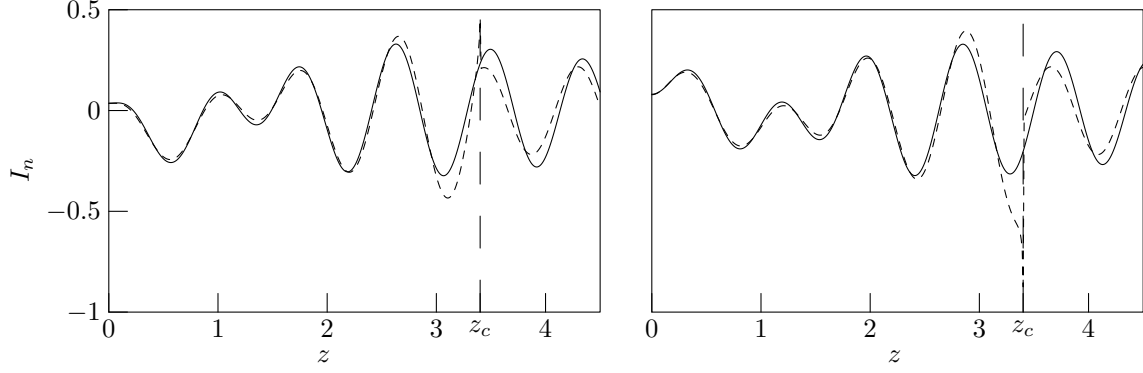


Figure 3: Numerical and asymptotic evaluation of I_n , $r = 1.125$, $k = 8$, $n = 1$: left hand side real, right hand side imaginary; solid line numerical evaluation, dashed line asymptotic, Equation 17

and:

$$I_m \sim (Q_{m+2}(k, r, z) - Q_m(k, r, z))/4, \quad (17)$$

where Q_{-m} and Q_{-m-2} are neglected since they have no stationary phase points and decay much faster than Q_m and Q_{m+2} .

Figure 3 compares the asymptotic approximation for $I_1(k, r, z)$ to a numerical evaluation of the integral. The cut-off value of z for $\gamma = 3/k$ is indicated and the real and imaginary parts of the integrals are plotted separately. As $z \rightarrow z_c$, the stationary phase approximation to Q_{m+2} breaks down and there is a resulting loss of accuracy. Away from this point, however, the approximation to I_m is accurate, on both sides of z_c .

The asymptotic analysis shows that the radiating line source modes, those with $m < k$ radiate efficiently into a region bounded by $\pm z_c(r, \gamma)$. Within that region, the result is accurate over the whole range from near to far field. This gives a second limit on the information available in the acoustic field.

3.3 Far field approximation

For completeness, we give a far field approximation of the line source radiation integral, valid outside the region covered by the asymptotic expansions of §3.2. Using the standard approximation, $R \approx R_0 + (r - r_2) \sin \phi$, $1/R \approx 1/R_0$ with $R_0^2 = r^2 + z^2$:

$$I_m \approx j^m \pi \frac{e^{jkR_0}}{R_0} \frac{m+1}{k \sin \phi} \left[\left(r + j \frac{(m+2)}{k \sin \phi} \right) J_{m+1}(k \sin \phi) - j J_m(k \sin \phi) \right] \quad (18)$$

4 Information in spinning sound fields

Summarizing the results of the previous section, the nature of a spinning sound field is seen to be determined by its wavenumber k and its relation to the set of modes contained in the

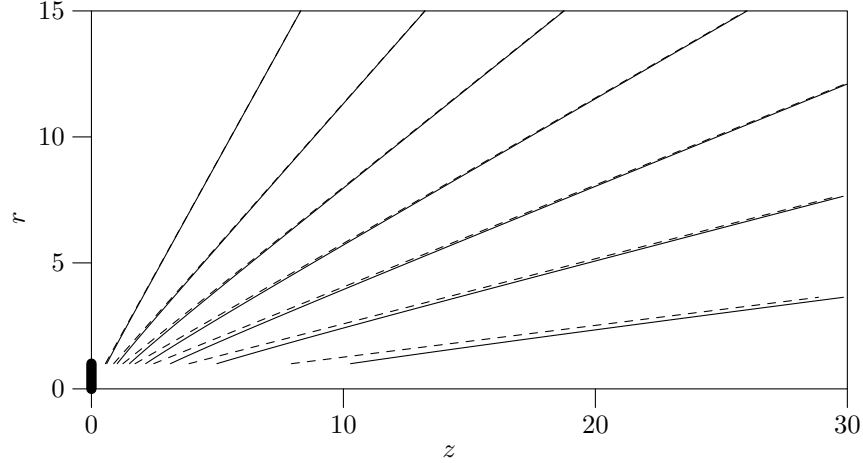


Figure 4: Cut-off lines for $\gamma = m/8$, $m = 1, 2, \dots, 6, 7$; exact solution solid; $z_c = \alpha r/\gamma$ dashed.

line source equivalent to the two-dimensional disk source. The first result, that line modes with $m > k$ generate exponentially small fields, implies that the acoustic field contains a limited amount of information about the source. Such a result has been derived previously by showing that the far field pressure is a band-limited Fourier transform of the line source strength [25], but this new result establishes a fundamental limit on the information radiated into the field, without recourse to a far-field approximation.

The second result, from the stationary phase analysis, shows that the modes which do radiate, those with $m < k$, are more efficient in some parts of the field than in others. The higher order radiating modes are detectable only near the source plane, with a lower radiation efficiency at larger z . The only mode which radiates efficiently over the whole field is the ‘plane’ mode $m = 0$.

The following sections discuss some implications of these findings for different problems.

4.1 Low speed rotors

One result which is of immediate interest is that, in some sense, low speed rotors have the same acoustic field. Given that for a system of radius a rotating at angular velocity Ω the non-dimensional wavenumber of the n th harmonic of the radiated field $k = n\Omega a/c = nM_t$, with M_t the source tip Mach number, low speed rotors will have $k < 1$ over the first few harmonics of the signal. This means that the sound field is dominated by the zero order line mode and any set of rotors of a given blade number operating at the same speed, whatever their blade geometry, will have the same acoustic field, to within a scaling factor.

4.2 Source identification

The original motivation for this work was the problem of identifying a rotating source. The results of §2 and §3 can be used to help show why this is a hard problem and to indicate

how it might best be approached.

The first obvious consequence of the result of §3.1 is that the acoustic field has a limited number of degrees of freedom. For a field of given wavenumber k , no more than k modes can be detected in the field, i.e. field has M degrees of freedom, with M the largest integer $M < k$. Attempting to identify sources using more than M degrees of freedom is inherently ill-conditioned because the modes with $m > k$ have decayed exponentially.

Secondly, the asymptotic analysis indicates where in the field it is possible to detect those modes which do radiate and gives a hint as to where microphones might be placed to best extract the information which is available.

4.3 Jet noise

In a recent paper [11], Jordan *et al.* examine noise production by a turbulent jet using proper orthogonal decomposition (POD) to perform a modal decomposition of the flow and a technique called MOD ('most observable decomposition') to decompose the acoustic far field. They find that while 350 flow modes are needed to account for 50% of the turbulent kinetic energy in the flow, only 24 modes are needed to account for 90% of the acoustic energy. If the jet is viewed as a distribution of disk sources along the jet axis, the results of this paper show that we should expect that only a small fraction of the modes will radiate noise and that in a complex source such as a jet, the bulk of the modes will have $m > k$ and will generate exponentially decaying fields. In a study of noise sources in a jet, Freund filters the source terms to leave "a set of modes capable of radiating to the far field", based on a wavenumber criterion, but he notes that "additional cancellation may occur due to the radial structure of the source" [26]. The analysis of the previous sections offers an approach for the identification of the radial terms which will give such cancellation.

5 Conclusions

An analysis of the radiation characteristics of spinning sources has been presented, based on the decomposition of the source into a set of exactly equivalent line sources. The results show that for any given wavenumber, there is a cut-off value of line mode order m , above which the radiated field decays exponentially. Below this cut-off, each mode radiates efficiently into a region bounded by rays from the source centre, with amplitude proportional to $k^{-1/2}$. These rays contract onto the source plane as the mode order is increased and disappear altogether when the order is equal to the wavenumber. The results have been used to explain the ill-conditioning of the source identification problem and some of the features of jet noise which have been noted in the literature. In future work, the analysis may be used to explain the structure of spinning sound fields, and to guide the development of acoustic systems for source identification.

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